OTHER COLLEGE

FYJC - MATHEMATICS & STATISTICS

PAPER - I

COMPOUND ANGLES

Compiled & Conducted @ JKSC

COMPOUND ANGLES - 5.5

MULTIPLE & SUBMULTIPLE ANGLES

sin	=	2 sin . cos
1 – cos	=	$2\sin^2$
1 + cos	=	$2\cos^2$
tan	=	$\dfrac{2\tan}{1-\tan^2}$

cos	=	$\cos^2 - \sin^2$
1 + sin	=	$(\cos^2 + \sin^2 + 2.\sin\cos)$
	=	$(\cos + \sin)^2$
1 – sin	=	$(\cos^2 + \sin^2 - 2.\sin\cos)$
	=	$(\cos - \sin)^2$

$\sin 3\theta$	=	$3\sin\theta - 4\sin^3\theta$
$\cos 3\theta$	=	$4\cos^3\theta - 3\cos\theta$
$\tan 3\theta$	=	$\dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

Q SET - 1

01 $\dfrac{\sin 2x}{1+\cos 2x} = \tan x$

02 . $\dfrac{1-\cos 2x}{\sin 2x} = \tan x$

03. $\dfrac{\sin 2\theta}{1-\cos 2\theta} = \cot\theta$

04. $\dfrac{\sin x}{1-\cos x} = \cot x$

05. $\sqrt{\dfrac{1-\cos 2x}{1+\cos 2x}} = \tan x$

06. $\sqrt{\dfrac{1+\cos A}{1-\cos A}} = \cot {}^{A}/_{2}$

07. $\dfrac{1+\cos A+\cos 2A}{\sin A+\sin 2A} = \cot A$

08. $\dfrac{1+\cos\theta+\sin\theta}{1-\cos\theta+\sin\theta} = \cot {}^{\theta}/_{2}$

Q SET - 2

01. $\sqrt{\dfrac{1-\sin 2A}{1+\sin 2A}} = \dfrac{1-\tan A}{1+\tan A}$

02. $\dfrac{\cos 2A}{1-\sin 2A} = \dfrac{1+\tan A}{1-\tan A}$

03. $\dfrac{\cos\theta}{1+\sin\theta} = \dfrac{1-\tan(\theta/2)}{1+\tan(\theta/2)}$

04. $\dfrac{\cos A}{1+\sin A} = \dfrac{\cot(A/2)-1}{\cot(A/2)+1}$

05. $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} = \tan\left(\dfrac{\pi}{4}+\dfrac{\theta}{2}\right)$

06. $\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right) = \frac{1}{\sec\theta - \tan\theta}$

Q SET - 3

01.

$\frac{\sin 16\theta}{\sin\theta} = 16\cos\theta \cdot \cos 2\theta \cdot \cos 4\theta \cdot \cos 8\theta$

02.

$\cos 5^\circ \cdot \cos 10^\circ \cdot \cos 20^\circ \cdot \cos 40^\circ = \frac{\cos 10^\circ}{16\ \sin 5^\circ}$

03.

$\cos 3^\circ \cdot \cos 6^\circ \cdot \cos 12^\circ \cdot \cos 24^\circ = \frac{\cos 42^\circ}{16\ \sin 3^\circ}$

04.

$\cos 7^\circ \cdot \cos 14^\circ \cdot \cos 28^\circ \cdot \cos 56^\circ = \frac{\sin 68^\circ}{16\cos 83^\circ}$

05. $\sqrt{2 + \sqrt{2 + 2\cos 4\theta}} = 2\cos\theta$

06. $\sqrt{2 + \sqrt{2 + 2\cos 2A}} = 2\cos {}^{A}/_{2}$

07. $\sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 8\theta}}} = 2\cos\theta$

08. $\frac{\sin 3A}{\cos A} + \frac{\cos 3A}{\sin A} = 2\cot 2A$

Q SET - 4

01.

$\cos 10^\circ \cdot \cos 30^\circ \cdot \cos 50^\circ \cdot \cos 70^\circ = 3/16$

02.

$\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ = 3/16$

03.

$4.\cos A \cdot \cos ({}^{\pi}/_{3} - A) \cdot \cos ({}^{\pi}/_{3} + A) = \cos 3A$

04.

$\sin 10^\circ \cdot \sin 30^\circ \cdot \sin 50^\circ \cdot \sin 70^\circ = 1/16$

05.

$\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 60^\circ \cdot \sin 80^\circ = 3/16$

06.

$\tan 20^\circ \cdot \tan 40^\circ \cdot \tan 60^\circ \cdot \tan 80^\circ = 3$

SOLUTION TO Q SET - 1

01 $\dfrac{\sin 2x}{1 + \cos 2x} = \tan x$

$$LHS = \frac{2\sin x \cdot \cos x}{2\cos^2 x}$$

$$= \frac{\sin x}{\cos x}$$

$$= \tan x = RHS$$

02 . $\dfrac{1 - \cos 2x}{\sin 2x} = \tan x$

$$LHS = \frac{2\sin^2 x}{2\sin x \cdot \cos x}$$

$$= \frac{\sin x}{\cos x}$$

$$= \tan x = RHS$$

03. $\dfrac{\sin 2\theta}{1 - \cos 2\theta} = \cot \theta$

$$LHS = \frac{2\sin \theta \cdot \cos \theta}{2\sin^2 \theta}$$

$$= \frac{\cos \theta}{\sin \theta}$$

$$= \cot \theta = RHS$$

04. $\dfrac{\sin x}{1 - \cos x} = \cot x$

$$LHS = \frac{2\sin {}^{x}/_{2} \cdot \cos {}^{x}/_{2}}{2\sin^2 {}^{x}/_{2}}$$

$$= \frac{\cos {}^{x}/_{2}}{\sin {}^{x}/_{2}}$$

$$= \tan {}^{x}/_{2} = RHS$$

05. $\sqrt{\dfrac{1 - \cos 2x}{1 + \cos 2x}} = \tan x$

$$LHS = \sqrt{\frac{2\sin^2 x}{2\cos^2 x}}$$

$$= \frac{\sin x}{\cos x}$$

$$= \tan x = RHS$$

06. $\sqrt{\dfrac{1 + \cos A}{1 - \cos A}} = \cot {}^{A}/_{2}$

$$LHS = \sqrt{\frac{2\cos^2 {}^{A}/_{2}}{2\sin^2 {}^{A}/_{2}}}$$

$$= \frac{\cos {}^{A}/_{2}}{\sin {}^{A}/_{2}}$$

$$= \cot {}^{A}/_{2} = RHS$$

07. $\dfrac{1 + \cos A + \cos 2A}{\sin A + \sin 2A} = \cot A$

$$LHS = \frac{1 + \cos 2A + \cos A}{\sin 2A + \sin A}$$

$$= \frac{2\cos^2 A + \cos A}{2\sin A \cdot \cos A + \sin A}$$

$$= \frac{\cos A(2\cos A + 1)}{\sin A\,(2\cos A + 1)}$$

$$= \frac{\cos A}{\sin A}$$

$$= \cot A = RHS$$

08. $$\frac{1 + \cos\theta + \sin\theta}{1 - \cos\theta + \sin\theta} = \cot \theta/2$$

$$\text{LHS} = \frac{1 + \cos\theta + \sin\theta}{1 - \cos\theta + \sin\theta}$$

$$= \frac{2\cos^2 \theta/2 + 2\sin \theta/2.\cos \theta/2}{2\sin^2 \theta/2 + 2\sin \theta/2.\cos \theta/2}$$

$$= \frac{2\cos \theta/2 \ (\cos \theta/2 + \sin \theta/2)}{2\sin \theta/2 \ (\cos \theta/2 + \sin \theta/2)}$$

$$= \frac{\cos \theta/2}{\sin \theta/2}$$

$$= \cot \theta/2 \quad = \quad \text{RHS}$$

SOLUTION TO Q SET - 2

01. $$\sqrt{\frac{1 - \sin 2A}{1 + \sin 2A}} = \frac{1 - \tan A}{1 + \tan A}$$

$$\text{LHS} = \sqrt{\frac{\cos^2 A + \sin^2 A - 2\sin A.\cos A}{\cos^2 A + \sin^2 A + 2\sin A.\cos A}}$$

$$= \sqrt{\frac{(\cos A - \sin A)^2}{(\cos A + \sin A)^2}}$$

$$= \frac{\cos A - \sin A}{\cos A + \sin A}$$

Dividing Numerator & Denominator by cos A

$$= \frac{\dfrac{\cos A - \sin A}{\cos A}}{\dfrac{\cos A + \sin A}{\cos A}}$$

$$= \frac{1 - \tan A}{1 + \tan A}$$

$$= \text{RHS}$$

02. $$\frac{\cos 2A}{1 - \sin 2A} = \frac{1 + \tan A}{1 - \tan A}$$

$$\text{LHS} = \frac{\cos^2 A - \sin^2 A}{\cos^2 A + \sin^2 A - 2\sin A.\cos A}$$

$$= \frac{(\cos A - \sin A).(\cos A + \sin A)}{(\cos A - \sin A)^2}$$

$$= \frac{\cos A + \sin A}{\cos A - \sin A}$$

Dividing Numerator & Denominator by cos A

$$= \frac{\dfrac{\cos A + \sin A}{\cos A}}{\dfrac{\cos A - \sin A}{\cos A}}$$

$$= \frac{1 + \tan A}{1 - \tan A} = \text{RHS}$$

03. $$\frac{\cos\theta}{1 + \sin\theta} = \frac{1 - \tan(\theta/2)}{1 + \tan(\theta/2)}$$

$$\text{LHS} = \frac{\cos^2 \theta/2 - \sin^2 \theta/2}{\cos^2 \theta/2 + \sin^2 \theta/2 + 2\sin \theta/2.\cos \theta/2}$$

$$= \frac{(\cos \theta/2 - \sin \theta/2).(\cos \theta/2 + \sin \theta/2)}{(\cos \theta/2 + \sin \theta/2)^2}$$

$$= \frac{\cos \theta/2 - \sin \theta/2}{\cos \theta/2 + \sin \theta/2}$$

Dividing Numerator & Denominator by cos A

$$= \frac{\dfrac{\cos \theta/2 - \sin \theta/2}{\cos \theta/2}}{\dfrac{\cos \theta/2 + \sin \theta/2}{\cos \theta/2}}$$

$$= \frac{1 - \tan \theta/2}{1 + \tan \theta/2} = \text{RHS}$$

04. $\dfrac{\cos A}{1+\sin A} = \dfrac{\cot(A/2)-1}{\cot(A/2)+1}$

$$\text{LHS} = \frac{\cos^2 {}^{A}/_{2} - \sin^2 {}^{A}/_{2}}{\cos^2 {}^{A}/_{2} + \sin^2 {}^{A}/_{2} + 2\sin {}^{A}/_{2}.\cos {}^{A}/_{2}}$$

$$= \frac{(\cos {}^{A}/_{2} - \sin {}^{A}/_{2}).(\cos {}^{A}/_{2} + \sin {}^{A}/_{2})}{(\cos {}^{A}/_{2} + \sin {}^{A}/_{2})^2}$$

$$= \frac{\cos {}^{A}/_{2} - \sin {}^{A}/_{2}}{\cos {}^{A}/_{2} + \sin {}^{A}/_{2}}$$

Dividing Numerator & Denominator by $\sin {}^{A}/_{2}$

$$= \frac{\dfrac{\cos {}^{A}/_{2} - \sin {}^{A}/_{2}}{\sin {}^{A}/_{2}}}{\dfrac{\cos {}^{A}/_{2} + \sin {}^{A}/_{2}}{\sin {}^{A}/_{2}}}$$

$$= \frac{\cot {}^{A}/_{2} - 1}{\cot {}^{A}/_{2} + 1} = \text{RHS}$$

05. $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} = \tan\left(\dfrac{\pi}{4} + \dfrac{\theta}{2}\right)$

$$\text{LHS} = \sqrt{\frac{\cos^2 {}^{\theta}/_{2} + \sin^2 {}^{\theta}/_{2} + 2\sin {}^{\theta}/_{2}.\cos {}^{\theta}/_{2}}{\cos^2 {}^{\theta}/_{2} + \sin^2 {}^{\theta}/_{2} - 2\sin {}^{\theta}/_{2}.\cos {}^{\theta}/_{2}}}$$

$$= \sqrt{\frac{(\cos {}^{\theta}/_{2} + \sin {}^{\theta}/_{2})^2}{(\cos {}^{\theta}/_{2} - \sin {}^{\theta}/_{2})^2}}$$

$$= \frac{\cos {}^{\theta}/_{2} + \sin {}^{\theta}/_{2}}{\cos {}^{\theta}/_{2} - \sin {}^{\theta}/_{2}}$$

Dividing Numerator & Denominator by $\cos {}^{\theta}/_{2}$

$$= \frac{\dfrac{\cos {}^{\theta}/_{2} + \sin {}^{\theta}/_{2}}{\cos {}^{\theta}/_{2}}}{\dfrac{\cos {}^{\theta}/_{2} - \sin {}^{\theta}/_{2}}{\cos {}^{\theta}/_{2}}}$$

$$= \frac{1 + \tan {}^{\theta}/_{2}}{1 - \tan {}^{\theta}/_{2}} = \text{RHS}$$

$$\text{RHS} = \tan({}^{\pi}/_{4} + {}^{\theta}/_{2})$$

$$= \frac{\tan {}^{\pi}/_{4} + \tan {}^{\theta}/_{2}}{1 - \tan {}^{\pi}/_{4}\tan {}^{\theta}/_{2}}$$

$$= \frac{1 + \tan {}^{\theta}/_{2}}{1 - \tan {}^{\theta}/_{2}}$$

LHS = RHS

06. $\tan\left(\dfrac{\pi}{4} + \dfrac{\theta}{2}\right) = \dfrac{1}{\sec\theta - \tan\theta}$

$$\text{RHS} = \frac{1}{\sec\theta - \tan\theta}$$

$$= \frac{1}{\dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}}$$

$$= \frac{\cos\theta}{1 - \sin\theta}$$

$$= \frac{\cos^2 {}^{\theta}/_{2} - \sin^2 {}^{\theta}/_{2}}{\cos^2 {}^{\theta}/_{2} + \sin^2 {}^{\theta}/_{2} - 2\sin {}^{\theta}/_{2}.\cos {}^{\theta}/_{2}}$$

$$= \frac{(\cos {}^{\theta}/_{2} - \sin {}^{\theta}/_{2}).(\cos {}^{\theta}/_{2} + \sin {}^{\theta}/_{2})}{(\cos {}^{\theta}/_{2} - \sin {}^{\theta}/_{2})^2}$$

$$= \frac{\cos {}^{\theta}/_{2} + \sin {}^{\theta}/_{2}}{\cos {}^{\theta}/_{2} - \sin {}^{\theta}/_{2}}$$

Dividing Numerator & Denominator by cos A

$$= \frac{\dfrac{\cos {}^{\theta}/_{2} + \sin {}^{\theta}/_{2}}{\cos {}^{\theta}/_{2}}}{\dfrac{\cos {}^{\theta}/_{2} - \sin {}^{\theta}/_{2}}{\cos {}^{\theta}/_{2}}}$$

$$= \frac{1 + \tan {}^{\theta}/_{2}}{1 - \tan {}^{\theta}/_{2}}$$

$$RHS = \tan\left({}^{\pi}/_{4} + {}^{\theta}/_{2}\right)$$

$$= \frac{\tan {}^{\pi}/_{4} + \tan {}^{\theta}/_{2}}{1 - \tan {}^{\pi}/_{4} \tan {}^{\theta}/_{2}}$$

$$= \frac{1 + \tan {}^{\theta}/_{2}}{1 - \tan {}^{\theta}/_{2}}$$

LHS = RHS

SOLUTION TO Q SET - 3

01.

$\sin 16\theta = 16 \cos\theta . \cos 2\theta . \cos 4\theta . \cos 8\theta$
$\sin\theta$

We Prove

$16 \sin\theta . \cos\theta . \cos 2\theta . \cos 4\theta . \cos 8\theta = \sin 16\theta$

LHS

$= 2.2.2.(2 \sin\theta \cos\theta). \cos 2\theta . \cos 4\theta . \cos 8\theta$

$= 2.2.(2.\sin 2\theta. \cos 2\theta) . \cos 4\theta . \cos 8\theta$

$= 2.(2.\sin 4\theta . \cos 4\theta) . \cos 8\theta$

$= 2.\sin 8\theta . \cos 8\theta$

$= \sin 16\theta \quad = RHS$

02.

$$\cos 5^\circ . \cos 10^\circ . \cos 20^\circ . \cos 40^\circ = \frac{\cos 10^\circ}{16 \sin 5^\circ}$$

We Prove

$16 \sin 5^\circ . \cos 5^\circ . \cos 10^\circ . \cos 20^\circ . \cos 40^\circ = \cos 10^\circ$

LHS

$= 16 \sin 5^\circ.\cos 5^\circ. \cos 10^\circ . \cos 20^\circ . \cos 40^\circ$

$= 2.2.2. \underline{2\sin 5^\circ.\cos 5^\circ} \cos 10^\circ.\cos 20^\circ.\cos 40^\circ$

$= 2.2. \underline{2.\sin 10^\circ \cos 10^\circ} \cos 20^\circ . \cos 40^\circ$

$= 2. \underline{2 \sin 20^\circ \cos 20^\circ} \cos 40^\circ$

$= 2 \sin 40^\circ \cos 40^\circ$

$= \sin 80^\circ$

$= \sin(90^\circ - 80^\circ)$

$= \cos 10^\circ$

03.

$$\cos 3^\circ . \cos 6^\circ . \cos 12^\circ . \cos 24^\circ = \frac{\cos 42^\circ}{16 \sin 3^\circ}$$

We Prove

$16 \sin 3^\circ . \cos 3^\circ . \cos 6^\circ . \cos 12^\circ . \cos 24^\circ = \cos 42^\circ$

LHS

$= 16 \sin 3^\circ.\cos 3^\circ. \cos 6^\circ . \cos 12^\circ . \cos 24^\circ$

$= 2.2.2. \underline{2\sin 3^\circ.\cos 3^\circ} \cos 6^\circ.\cos 12^\circ. \cos 24^\circ$

$= 2.2. \underline{2.\sin 6^\circ \cos 6^\circ} \cos 12^\circ . \cos 24^\circ$

$= 2. \underline{2 \sin 12^\circ \cos 12^\circ} \cos 24^\circ$

$= 2 \sin 24^\circ \cos 24^\circ$

$= \sin 48^\circ$

$= \sin(90^\circ - 42^\circ)$

$= \cos 42^\circ$

04.

$$\cos 7^\circ . \cos 14^\circ . \cos 28^\circ . \cos 56^\circ = \frac{\sin 68^\circ}{16\cos 83^\circ}$$

We Prove

$16\cos 83^\circ . \cos 7^\circ . \cos 14^\circ . \cos 28^\circ . \cos 56^\circ = \sin 68^\circ$

LHS

$= 16\cos 83^\circ.\cos 7^\circ. \cos 14^\circ . \cos 18^\circ . \cos 56^\circ$

$= 16 \sin 7^\circ.\cos 7^\circ. \cos 14^\circ . \cos 18^\circ . \cos 56^\circ$

$= 2.2.2.2\sin 7^\circ.\cos 7^\circ \cos 14^\circ.\cos 28^\circ.\cos 56^\circ$

$= 2.2.\ 2.\sin 14^\circ \cos 14^\circ \cos 28^\circ.\cos 56^\circ$

$= 2.\ 2\sin 28^\circ \cos 28^\circ\ \cos 56^\circ$

$= 2\sin 56^\circ\ \cos 56^\circ$

$= \sin 112^\circ$

$= \sin(180^\circ - 68^\circ)$

$= \sin 68^\circ$

05.

$$\sqrt{2+\sqrt{2 + 2\cos 4\theta}} = 2\cos\theta$$

LHS

$= \sqrt{2+\sqrt{2(1+\cos 4\theta)}}$

$= \sqrt{2+\sqrt{2\,.\,2\cos^2 2\theta}}$

$= \sqrt{2+\sqrt{4\cos^2 2\theta}}$

$= \sqrt{2+2\cos 2\theta}$

$= \sqrt{2(1+\cos 2\theta)}$

$= \sqrt{2\,.\,2\cos^2\theta}$

$= \sqrt{4\cos^2\theta}$

$= 2\cos\theta$

$=$ RHS

06.

$$\sqrt{2+\sqrt{2 + 2\cos 2A}} = 2\cos {}^{A}/_{2}$$

LHS

$= \sqrt{2+\sqrt{2(1+\cos 2A)}}$

$= \sqrt{2+\sqrt{2\,.\,2\cos^2 A}}$

$= \sqrt{2+\sqrt{4\cos^2 A}}$

$= \sqrt{2+2\cos A}$

$= \sqrt{2(1+\cos A)}$

$= \sqrt{2\,.\,2\cos^2 {}^{A}/_{2}}$

$= \sqrt{4\cos^2 {}^{A}/_{2}}$

$= 2\cos {}^{A}/_{2}$

$=$ RHS

07.

$$\sqrt{2+\sqrt{2+\sqrt{2 + 2\cos 8\theta}}} = 2\cos\theta$$

LHS

$= \sqrt{2+\sqrt{2+\sqrt{2(1+\cos 8\theta)}}}$

$= \sqrt{2+\sqrt{2+\sqrt{2\,.\,2\cos^2 4\theta}}}$

$= \sqrt{2+\sqrt{2+\sqrt{4\cos^2 4\theta}}}$

$= \sqrt{2+\sqrt{2+2\cos 4\theta}}$

$$= \sqrt{2 + \sqrt{2\,(1 + \cos 4\theta)}}$$

$$= \sqrt{2 + \sqrt{2\,.2 \cos^2 2\theta}}$$

$$= \sqrt{2 + \sqrt{4 \cos^2 2\theta}}$$

$$= \sqrt{2 + 2 \cos 2\theta}$$

$$= \sqrt{2\,(1 + \cos 2\theta)}$$

$$= \sqrt{2\,.2 \cos^2 \theta}$$

$$= \sqrt{4 \cos^2 \theta}$$

$$= 2 \cos \theta$$

08. $\dfrac{\sin 3A}{\cos A} + \dfrac{\cos 3A}{\sin A} = 2 \cot 2A$

LHS

$$= \frac{\sin 3A \,.\, \sin A + \cos 3A \,.\cos A}{\sin A \,.\, \cos A}$$

$$= \frac{\cos 3A \,.\, \cos A + \sin 3A \,.\, \sin A}{\sin A \,.\, \cos A}$$

$$= \frac{\cos (3A - A)}{\sin A \,.\, \cos A}$$

$$= \frac{\cos 2A}{\sin A \,.\, \cos A}$$

$$= \frac{2 \cos 2A}{2 \sin A \,.\, \cos A}$$

$$= \frac{2 \cos 2A}{\sin 2A}$$

$$= 2 \cot 2A \quad = \quad \text{RHS}$$

SOLUTION TO Q SET - 4

01. $\cos 10^\circ \,.\, \cos 30^\circ \,.\, \cos 50^\circ \,.\, \cos 70^\circ = 3/16$

LHS $= \cos 10^\circ \,.\, \cos 30^\circ \,.\, \cos 50^\circ \,.\, \cos 70^\circ$

$$= \frac{\sqrt{3}}{2} \,.\, \cos 10^\circ \,.\, \cos (60 - 10)^\circ \,.\, \cos (60 + 10)^\circ$$

$$= \frac{\sqrt{3}}{2} \,.\, \cos 10^\circ \,.\, (\cos 60 \cos 10 + \sin 60 \sin 10) \,.\, (\cos 60 \cos 10 - \sin 60 \sin 10)$$

$$= \frac{\sqrt{3}}{2} \,.\, \cos 10^\circ \,.\, \left(\frac{1}{2} \cos 10 + \frac{\sqrt{3}}{2} \sin 10\right) . \left(\frac{1}{2} \cos 10 - \frac{\sqrt{3}}{2} \sin 10\right)$$

$$= \frac{\sqrt{3}}{2} \,.\, \cos 10^\circ \,.\, \left(\frac{1}{4} \cos^2 10 - \frac{3}{4} \sin^2 10\right)$$

$$= \frac{\sqrt{3}}{8} \,.\, \cos 10^\circ \,.\, \left(\cos^2 10 - 3 \sin^2 10\right)$$

$$= \frac{\sqrt{3}}{8} \,.\, \cos 10^\circ \,.\, \left(\cos^2 10 - 3\,(1 - \cos^2 10)\right)$$

$$= \frac{\sqrt{3}}{8} . \cos 10^\circ . \left(\cos^2 10 - 3 + 3\cos^2 10\right)$$

$$= \frac{\sqrt{3}}{8} . \cos 10^\circ . (4\cos^2 10 - 3)$$

$$= \frac{\sqrt{3}}{8} . (4\cos^3 10 - 3 \cos 10^\circ)$$

$4\cos^3\theta - 3\cos \theta = \cos 3\theta$

$$= \frac{\sqrt{3}}{8} . \cos 30^\circ$$

$$= \frac{\sqrt{3}}{8} . \frac{\sqrt{3}}{2}$$

$$= \frac{3}{16}$$

02. $\cos 20^\circ . \cos 40^\circ . \cos 60^\circ . \cos 80^\circ = 3/16$

LHS $= \cos 20^\circ . \cos 40^\circ . \cos 60^\circ . \cos 80^\circ$

$$= \frac{1}{2} . \cos 20^\circ . \cos (60 - 20)^\circ . \cos (60 + 20)^\circ$$

$$= \frac{1}{2} . \cos 20^\circ . (\cos 60 \cos 20 + \sin 60 \sin 20) . (\cos 60 \cos 20 - \sin 60 \sin 20)$$

$$= \frac{1}{2} . \cos 20^\circ . \left(\frac{1}{2}\cos 20 + \frac{\sqrt{3}}{2}\sin 20\right) . \left(\frac{1}{2}\cos 20 - \frac{\sqrt{3}}{2}\sin 20\right)$$

$$= \frac{1}{2} . \cos 20^\circ . \left(\frac{1}{4}\cos^2 20 - \frac{3}{4}\sin^2 20\right)$$

$$= \frac{1}{8} . \cos 20^\circ . \left(\cos^2 20 - 3 \sin^2 20\right)$$

$$= \frac{1}{8} . \cos 20^\circ . \left(\cos^2 20 - 3 (1 - \cos^2 20) \right)$$

$$= \frac{1}{8} . \cos 20^\circ . \left(\cos^2 20 - 3 + 3\cos^2 20\right)$$

$$= \frac{1}{8} . \cos 20^\circ . (4\cos^2 20 - 3)$$

$$= \frac{1}{8} \cdot (4\cos^3 20 \; - \; 3 \cos 20^{\circ})$$

$$4\cos^3\theta - 3\cos\theta \;=\; \cos 3\theta$$

$$= \frac{1}{8} \cdot \cos(2 \times 20^{\circ})$$

$$= \frac{1}{8} \cdot \cos 60^{\circ}$$

$$= \frac{1}{8} \cdot \frac{1}{2}$$

$$= \frac{1}{16}$$

03. $4.\cos A \cdot \cos(^{\pi}/_{3} - A) \cdot \cos(^{\pi}/_{3} + A) = \cos 3A$

$$\text{LHS} = 4.\cos A \cdot \cos(60 - A) \cdot \cos(60 + A)$$

$$= 4.\cos A \cdot (\cos 60 \cos A + \sin 60 \sin A) \cdot (\cos 60 \cos A - \sin 60 \sin A)$$

$$= 4 \cdot \cos A \cdot \left(\frac{1}{2}\cos A + \frac{\sqrt{3}}{2}\sin A\right) \cdot \left(\frac{1}{2}\cos A - \frac{\sqrt{3}}{2}\sin A\right)$$

$$= 4.\cos A \cdot \left(\frac{1}{4}\cos^2 A - \frac{3}{4}\sin^2 A\right)$$

$$= \cos A \cdot \left[\cos^2 A - 3(1 - \cos^2 A)\right]$$

$$= \cos A \cdot \left[\cos^2 A - 3 + 3\cos^2 A\right]$$

$$= \cos A \cdot \left[4\cos^2 A - 3\right]$$

$$= 4\cos^3 A - 3\cos A$$

$$= \cos 3A$$

04. $\sin 10^{\circ} \cdot \sin 30^{\circ} \cdot \sin 50^{\circ} \cdot \sin 70^{\circ} \quad = \quad 1/16$

$$\text{LHS} = \sin 10^{\circ} \cdot \sin 30^{\circ} \cdot \sin 50^{\circ} \cdot \sin 70^{\circ}$$

$$= \frac{1}{2} \cdot \sin 10^{\circ} \cdot \sin(60 - 10)^{\circ} \cdot \sin(60 + 10)^{\circ}$$

$$= \frac{1}{2} \cdot \sin 10^{\circ} \cdot (\sin 60 \cos 10 - \cos 60 \sin 10) \cdot (\sin 60 \cos 10 + \cos 60 \sin 10)$$

$$= \frac{1}{2} . \quad \sin 10^{o} . \quad \left(\frac{\sqrt{3}}{2}\cos 10 + \frac{1}{2}\sin 10\right) . \left(\frac{\sqrt{3}}{2}\cos 10 + \frac{1}{2}\sin 10\right)$$

$$= \frac{1}{2} . \quad \sin 10^{o} . \quad \left(\frac{3}{4}\cos^2 10 - \frac{1}{4}\sin^2 10\right)$$

$$= \frac{1}{8} . \quad \sin 10^{o} . \quad \left(3 \cos^2 10 - \sin^2 10\right)$$

$$= \frac{1}{8} . \quad \sin 10^{o} . \quad \left(3\,(1 - \sin^2 10) - \sin^2 10\right)$$

$$= \frac{1}{8} . \quad \sin 10^{o} . \quad \left(3 - 3\sin^2 10 - \sin^2 10\right)$$

$$= \frac{1}{8} . \quad \sin 10^{o} . (3 - 4\sin^2 10)$$

$$= \frac{1}{8} . \quad (3 \sin 10^{o} - 4 \sin^3 10)$$

$$= \frac{1}{8} . \quad \sin (3 \times 10^{o})$$

$$= \frac{1}{8} . \sin 30$$

$$= \frac{1}{8} . \frac{1}{2}$$

$$= \frac{1}{16}$$

05. $\sin 20^{o} . \sin 40^{o} . \sin 60^{o} . \sin 80^{o} = 3/16$

LHS $= \sin 20^{o} . \sin 40^{o} . \sin 60^{o} . \sin 80^{o}$

$$= \frac{\sqrt{3}}{2} . \quad \sin 20^{o} . \quad \sin (60 - 20)^{o} . \sin (60 + 20)^{o}$$

$$= \frac{\sqrt{3}}{2} . \quad \sin 20^{o} . \quad (\sin 60 \cos 20 - \cos 60 \sin 20) . (\sin 60 \cos 20 + \cos 60 \sin 20)$$

$$= \frac{\sqrt{3}}{2} . \quad \sin 20^{o} . \quad \left(\frac{\sqrt{3}}{2}\cos 20 + \frac{1}{2}\sin 20\right) . \left(\frac{\sqrt{3}}{2}\cos 20 + \frac{1}{2}\sin 20\right)$$

$$= \frac{\sqrt{3}}{2} . \quad \sin 20^{o} . \quad \left(\frac{3}{4}\cos^2 20 - \frac{1}{4}\sin^2 20\right)$$

$$= \frac{\sqrt{3}}{8} . \quad \sin 20^{o} . \quad \left(3 \cos^2 20 - \sin^2 20\right)$$

$$= \frac{\sqrt{3}}{8} . \quad \sin 20^{o} . \quad \left(3\,(1 - \sin^2 20) - \sin^2 20\right)$$

$$= \frac{\sqrt{3}}{8} . \quad \sin 20^{o} . \quad \left(3 - 3\sin^2 20 - \sin^2 20\right)$$

$= \frac{\sqrt{3}}{8} . \quad \sin 20^\circ . (3 - 4\sin^2 20)$

$= \frac{\sqrt{3}}{8} . \quad (3 \sin 20^\circ - 4 \sin^3 20)$

$3\sin\theta - 4\sin^3\theta = \sin 3\theta$

$= \frac{\sqrt{3}}{8} . \quad \sin (3 \times 20^\circ)$

$= \frac{\sqrt{3}}{8} . \quad \sin 30^\circ$

$= \frac{\sqrt{3}}{8} . \frac{1}{2}$

$= \frac{3}{16}$

06. $4.\sin A . \sin ({}^{\pi}/_{3} - A) . \sin ({}^{\pi}/_{3} + A) = \sin 3A$

LHS $= 4.\sin A . \sin (60 - A) . \sin (60 + A)$

$= 4.\sin A . (\sin 60 \cos A - \cos 60 \sin A) . (\sin 60 \cos A + \cos 60 \sin A)$

$= 4 . \sin A . \left(\frac{\sqrt{3}}{2} \cos A - \frac{1}{2} \sin A\right) . \left(\frac{\sqrt{3}}{2} \cos A + \frac{1}{2} \sin A\right)$

$= 4.\sin A . \left(\frac{3}{4} \cos^2 A - \frac{1}{4} \sin^2 A\right)$

$= \sin A . \left[3(1 - \sin^2 A) - \sin^2 A\right]$

$= \sin A . \left[3 - 3 \sin^2 A - \sin^2 A\right]$

$= \sin A . \left[3 - 4 \sin^2 A\right]$

$= 3 \sin A - 4 \sin^3 A$

$= \sin 3A$

07. $\tan 20^\circ . \tan 40^\circ . \tan 60^\circ . \tan 80^\circ = 3$

LHS $= \tan 20^\circ . \tan 40^\circ . \tan 60^\circ . \tan 80^\circ$

$= \sqrt{3} \tan 20^\circ . \tan (60 - 20)^\circ . \tan (60 + 20)^\circ$

$$= \sqrt{3}\tan 20^\circ \,.\, \left(\frac{\tan 60 - \tan 20}{1 + \tan 60 \,.\, \tan 20}\right) . \left(\frac{\tan 60 + \tan 20}{1 - \tan 60 \,.\, \tan 20}\right)$$

$$= \sqrt{3}\tan 20^\circ \,.\, \left(\frac{\sqrt{3} - \tan 20}{1 + \sqrt{3} \,.\, \tan 20}\right)\left(\frac{\sqrt{3} + \tan 20}{1 - \sqrt{3} \,.\, \tan 20}\right)$$

$$= \sqrt{3}\tan 20^\circ \,.\, \left(\frac{3 - \tan^2 20}{1 - 3\tan^2 20}\right)$$

$$= \sqrt{3}\left(\frac{3\tan 20^\circ - \tan^3 20^\circ}{1 - 3\tan^2 20^\circ}\right)$$

$$\boxed{\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} = \tan 3\theta}$$

$$= \sqrt{3}\ \tan(3 \times 20)^\circ$$

$$= \sqrt{3}\ \tan 60^\circ$$

$$= \sqrt{3} \,.\, \sqrt{3}$$

$$= 3$$

= RHS

COMPOUND ANGLES - 5.6

FACTORISE & DEFACTORISE

FACTORIZATION FORMULAE

$$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$$

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right).\sin\left(\frac{C-D}{2}\right)$$

$$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$$

$$\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right).\sin\left(\frac{C-D}{2}\right)$$

DE FACTORIZATION FORMULAE

$$2.\sin A . \cos B = \sin(A+B) + \sin(A-B)$$

$$2.\cos A . \sin B = \sin(A+B) - \sin(A-B)$$

$$2.\cos A . \cos B = \cos(A+B) + \cos(A-B)$$

$$2.\sin A . \sin B = \cos(A-B) - \cos(A+B)$$

Q SET - 1

01. $$\frac{\cos 4\theta + \cos 2\theta}{\sin 4\theta - \sin 2\theta} = \cot\theta$$

02. $$\frac{\sin 5A + \sin 3A}{\cos 5A + \cos 3A} = \tan 4A$$

03. $$\frac{\sin 10\theta - \sin 2\theta}{\cos 2\theta - \cos 10\theta} = \cot 6\theta$$

04. $$\frac{\sin 8x + \sin 2x}{\cos 2x - \cos 8x} = \cot 3x$$

05. $$\frac{\cos 3\theta - \cos 11\theta}{\sin 11\theta - \sin 3\theta} = \tan 7\theta$$

06. $$\frac{\sin 2\alpha + \sin 2\beta}{\sin 2\alpha - \sin 2\beta} = \frac{\tan(\alpha+\beta)}{\tan(\alpha-\beta)}$$

07. $$\frac{\sin 2\alpha + \sin 2\beta}{\cos 2\alpha - \cos 2\beta} = \cot(\beta-\alpha)$$

08. $$\frac{\cos(7x-5y) + \cos(7y-5x)}{\sin(7x-5y) + \sin(7y-5x)} = \cot(x+y)$$

Q SET - 2

01. $$\frac{\sin A + \sin 4A + \sin 7A}{\cos A + \cos 4A + \cos 7A} = \tan 4A$$

02. $$\frac{\sin A + \sin 5A + \sin 9A}{\cos A + \cos 5A + \cos 9A} = \tan 5A$$

03. $$\frac{\sin 2\alpha + \sin 5\alpha - \sin\alpha}{\cos 2\alpha + \cos 5\alpha + \cos\alpha} = \tan 2\alpha$$

04. $$\frac{\sin 2\theta + 2.\sin 4\theta + \sin 6\theta}{\sin\theta + 2.\sin 3\theta + \sin 5\theta} = \cos\theta + \sin\theta . \cot 3\theta$$

05. $$\frac{\sin 3\theta + 2.\sin 5\theta + \sin 7\theta}{\sin\theta + 2.\sin 3\theta + \sin 5\theta} = \cos 2\theta + \sin 2\theta \,.\, \cot 3\theta$$

06. $$\frac{\cos 3A - 2.\cos 5A + \cos 7A}{\cos A - 2.\cos 3A + \cos 5A} = \cos 2A - \sin 2A \,.\, \tan 3A$$

07. $$\frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \operatorname{cosec} 2x - \cot 2x$$

Q SET - 3

01.
$$\frac{\sin A + \sin 2A + \sin 3A + \sin 4A}{\cos A + \cos 2A - \cos 3A - \cos 4A} = \cot A$$

02.
$$\frac{\sin x - \sin 3x + \sin 5x - \sin 7x}{\cos x - \cos 3x - \cos 5x + \cos 7x} = \cot 2x$$

03.
$$\frac{\sin x - \sin 5x + \sin 9x - \sin 13x}{\cos x - \cos 5x - \cos 9x + \cos 13x} = \cot 4x$$

04.
if $\sin 2x + \sin 6x = \cos 2x + \cos 6x$
Show :
either $\tan 4x = 1$ OR $\cos 2x = 0$

05.
$\sin A + \sin 2A + \sin 3A = \cos A + \cos 2A + \cos 3A$
Show :
either $\tan 2A = 1$ OR $\cos A = -1/2$

06.
$\sin 10^\circ + \sin 50^\circ - \sin 80^\circ + \sin 140^\circ = \sqrt{2} \,.\, \sin 25^\circ$

07. $\cos 40^\circ + \cos 50^\circ + \cos 70^\circ + \cos 80^\circ$

$= \cos 20^\circ + \cos 10^\circ$

Q SET - 4

01.
$$\frac{\sin 3A \,.\, \cos 4A - \sin A \,.\, \cos 2A}{\sin A \,.\, \sin 4A + \cos A \,.\, \cos 6A} = \tan 2A$$

02.
$$\frac{\sin 8\theta.\cos\theta - \sin 6\theta.\cos 3\theta}{\cos 2\theta.\cos\theta - \cos 3\theta.\cos 4\theta} = \cot 5\theta$$

03.
$$\frac{\sin 3\theta.\cos 5\theta - \sin\theta.\cos 7\theta}{\sin\theta.\sin 7\theta - \cos 3\theta.\cos 5\theta} = \tan 2\theta$$

04.
$$\frac{\cos 3A \,.\, \sin 9A - \sin A \,.\, \cos 5A}{\cos A \,.\, \cos 5A - \sin 3A \,.\, \sin 9A} = \tan 8A$$

Q SET - 5

01.
$\cos^2 x + \cos^2(x + 120) + \cos^2(x - 120) = 3/2$

02.
$\sin^2\theta + \sin^2(120 + \theta) + \sin^2(120 - \theta) = 3/2$

SOLUTION TO Q SET - 1

01. $\dfrac{\cos 4\theta + \cos 2\theta}{\sin 4\theta - \sin 2\theta} = \cot\theta$

LHS

$$= \frac{2\cos\left(\frac{4\theta+2\theta}{2}\right).\cos\left(\frac{4\theta-2\theta}{2}\right)}{2\cos\left(\frac{4\theta+2\theta}{2}\right).\sin\left(\frac{4\theta-2\theta}{2}\right)}$$

$$= \frac{\cancel{2\cos 3\theta}\,.\cos\theta}{\cancel{2\cos 3\theta}\,.\sin\theta}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$$= \cot\theta$$

02. $\dfrac{\sin 5A + \sin 3A}{\cos 5A + \cos 3A} = \tan 4A$

LHS

$$= \frac{2\sin\left(\frac{5A+3A}{2}\right).\cos\left(\frac{5A-3A}{2}\right)}{2\cos\left(\frac{5A+3A}{2}\right).\cos\left(\frac{5A-3A}{2}\right)}$$

$$= \frac{2\sin 4A\,.\cancel{\cos A}}{2\cos 4A\,.\cancel{\cos A}}$$

$$= \frac{\sin 4A}{\cos 4A}$$

$$= \tan 4A$$

03. $\dfrac{\sin 10\theta - \sin 2\theta}{\cos 2\theta - \cos 10\theta} = \cot 6\theta$

LHS

$$= \frac{2\cos\left(\frac{10\theta+2\theta}{2}\right)\sin\left(\frac{10\theta-2\theta}{2}\right)}{-\,2\sin\left(\frac{2\theta+10\theta}{2}\right).\sin\left(\frac{2\theta-10\theta}{2}\right)}$$

$$= \frac{2\cos 6\theta\,.\sin 4\theta}{-\,2\sin 6\theta\,.\sin(-4\theta)}$$

$$= \frac{2\cos 6\theta\,.\cancel{\sin 4\theta}}{2\sin 6\theta\,.\cancel{\sin 4\theta}}$$

$$= \frac{\cos 6\theta}{\sin 6\theta}$$

$$= \cot 6\theta$$

04. $\dfrac{\sin 8x + \sin 2x}{\cos 2x - \cos 8x} = \cot 3x$

LHS

$$= \frac{2\sin\left(\frac{8x+2x}{2}\right)\cos\left(\frac{8x-2x}{2}\right)}{-\,2\sin\left(\frac{2x+8x}{2}\right).\sin\left(\frac{2x-8x}{2}\right)}$$

$$= \frac{2\sin 5x\,.\cos 3x}{-\,2\sin 5x\,.\sin(-3x)}$$

$$= \frac{2\cancel{\sin 5x}\,.\cos 3x}{2\cancel{\sin 5x}\,.\sin 3x}$$

$$= \frac{\cos 3x}{\sin 3x}$$

$$= \cot 3x$$

05. $\dfrac{\cos 3\theta - \cos 11\theta}{\sin 11\theta - \sin 3\theta} = \tan 7\theta$

LHS

$$= \frac{2\sin\left(\frac{3\theta+11\theta}{2}\right)\sin\left(\frac{3\theta-11\theta}{2}\right)}{-\,2\cos\left(\frac{11\theta+3\theta}{2}\right).\sin\left(\frac{11\theta-3\theta}{2}\right)}$$

$$= \frac{-\,2\sin 7\theta\,.\sin 4\theta}{-\,2\sin 7\theta\,.\sin 4\theta}$$

$$= \frac{2 \sin 7\theta \,.\, \cancel{\sin 4\theta}}{2 \cos 7\theta \,.\, \cancel{\sin 4\theta}}$$

$$= \frac{\sin 7\theta}{\cos 7\theta}$$

$$= \tan 7\theta$$

06. $\dfrac{\sin 2\alpha + \sin 2\beta}{\sin 2\alpha - \sin 2\beta} = \dfrac{\tan(\alpha+\beta)}{\tan(\alpha-\beta)}$

LHS

$$= \frac{2 \sin\left(\frac{2\alpha+2\beta}{2}\right).\cos\left(\frac{2\alpha-2\beta}{2}\right)}{2\cos\left(\frac{2\alpha+2\beta}{2}\right).\sin\left(\frac{2\alpha-2\beta}{2}\right)}$$

$$= \frac{2 \sin(\alpha+\beta) \,.\, \cos(\alpha-\beta)}{2\cos(\alpha+\beta) \,.\, \sin(\alpha-\beta)}$$

$$= \tan(\alpha+\beta) \,.\, \cot(\alpha-\beta)$$

$$= \frac{\tan(\alpha+\beta)}{\tan(\alpha-\beta)}$$

07. $\dfrac{\sin 2\alpha + \sin 2\beta}{\cos 2\alpha - \cos 2\beta} = \cot(\beta-\alpha)$

LHS

$$= \frac{2 \sin\left(\frac{2\alpha+2\beta}{2}\right).\cos\left(\frac{2\alpha-2\beta}{2}\right)}{-2\sin\left(\frac{2\alpha+2\beta}{2}\right).\sin\left(\frac{2\alpha-2\beta}{2}\right)}$$

$$= \frac{2 \sin(\alpha+\beta).\cos(\alpha-\beta)}{-2\sin(\alpha+\beta) \,.\, \sin(\alpha-\beta)}$$

$$= -\cot(\alpha-\beta)$$

$$= \cot(\beta-\alpha)$$

08. $\dfrac{\cos(7x-5y) + \cos(7y-5x)}{\sin(7x-5y) + \sin(7y-5x)} = \cot(x+y)$

$$= \frac{2\cos\left(\frac{7x-5y+7y-5x}{2}\right).\cos\left(\frac{7x-5y-7y+5x}{2}\right)}{2\sin\left(\frac{7x-5y+7y-5x}{2}\right).\cos\left(\frac{7x-5y-7y+5x}{2}\right)}$$

$$= \frac{2\cos\left(\frac{2x+2y}{2}\right)\cos\left(\frac{12x-12y}{2}\right)}{2\sin\left(\frac{2x+2y}{2}\right)\cos\left(\frac{12x-12y}{2}\right)}$$

$$= \frac{2\cos(x+y) \,.\, \cos(x-y)}{2\sin(x+y) \,.\cos(x-y)}$$

$$= \frac{\cos(x+y)}{\sin(x+y)}$$

$$= \cot(x+y)$$

SOLUTION TO Q SET - 2

01. $\dfrac{\sin A + \sin 4A + \sin 7A}{\cos A + \cos 4A + \cos 7A} = \tan 4A$

LHS

$$= \frac{\sin 7A + \sin A + \sin 4A}{\cos 7A + \cos A + \cos 4A}$$

$$= \frac{2\sin\left(\frac{7A+A}{2}\right).\cos\left(\frac{7A-A}{2}\right) + \sin 4A}{2\cos\left(\frac{7A+A}{2}\right).\cos\left(\frac{7A-A}{2}\right) + \cos 4A}$$

$$= \frac{2\sin 4A \,.\, \cos 3A + \sin 4A}{2\cos 4A \,.\cos 3A + \cos 4A}$$

$$= \frac{\sin 4A\,\cancel{(2\cos 3A + 1)}}{\cos 4A\,\cancel{(2\cos 3A + 1)}}$$

$$= \tan 4A$$

02. $\dfrac{\sin A + \sin 5A + \sin 9A}{\cos A + \cos 5A + \cos 9A} = \tan 5A$

LHS

$$= \frac{\sin 9A + \sin A + \sin 5A}{\cos 9A + \cos A + \cos 5A}$$

$$= \frac{2\sin\left(\frac{9A + A}{2}\right).\cos\left(\frac{9A - A}{2}\right) + \sin 5A}{2\cos\left(\frac{9A + A}{2}\right).\cos\left(\frac{9A - A}{2}\right) + \cos 5A}$$

$$= \frac{2\sin 5A . \cos 4A + \sin 5A}{2\cos 5A . \cos 4A + \cos 5A}$$

$$= \frac{\sin 5A\,\cancel{(2\cos 4A + 1)}}{\cos 5A\,\cancel{(2\cos 4A + 1)}}$$

$$= \tan 5A$$

03. $\dfrac{\sin 2\alpha + \sin 5\alpha - \sin\alpha}{\cos 2\alpha + \cos 5\alpha + \cos\alpha} = \tan 2\alpha$

LHS

$$= \frac{\sin 2\alpha + \sin 5\alpha - \sin\alpha}{\cos 2\alpha + \cos 5\alpha + \cos\alpha}$$

$$= \frac{\sin 2\alpha + 2\cos\left(\frac{5\alpha + \alpha}{2}\right).\sin\left(\frac{5\alpha - \alpha}{2}\right)}{\cos 2\alpha + 2\cos\left(\frac{5\alpha + \alpha}{2}\right).\cos\left(\frac{5\alpha - \alpha}{2}\right)}$$

$$= \frac{\sin 2\alpha + 2\cos 4\alpha . \sin 2\alpha}{\cos 2\alpha + 2\cos 4\alpha . \cos 2\alpha}$$

$$= \frac{\sin 2\alpha\,\cancel{(1 + 2\cos 4\alpha)}}{\cos 2\alpha\,\cancel{(1 + 2\cos 4\alpha)}}$$

$$= \frac{\sin 2\alpha}{\cos 2\alpha}$$

$$= \tan 2\alpha$$

04. $\dfrac{\sin 2\theta + 2.\sin 4\theta + \sin 6\theta}{\sin\theta + 2.\sin 3\theta + \sin 5\theta} = \cos\theta + \sin\theta . \cot 3\theta$

LHS

$$= \frac{\sin 6\theta + \sin 2\theta + 2.\sin 4\theta}{\sin 5\theta + \sin\theta + 2.\sin 3\theta}$$

$$= \frac{2\sin\left(\frac{6\theta + 2\theta}{2}\right).\cos\left(\frac{6\theta - 2\theta}{2}\right) + 2.\sin 4\theta}{2\sin\left(\frac{5\theta + \theta}{2}\right).\cos\left(\frac{5\theta - \theta}{2}\right) + 2.\sin 3\theta}$$

$$= \frac{2\sin 4\theta . \cos 2\theta + 2.\sin 4\theta}{2\sin 3\theta . \cos 2\theta + 2.\sin 3\theta}$$

$$= \frac{2\sin 4\theta . \cancel{(\cos 2\theta + 1)}}{2\sin 3\theta . \cancel{(\cos 2\theta + 1)}}$$

$$= \frac{\sin 4\theta}{\sin 3\theta}$$

$$= \frac{\sin(3\theta + \theta)}{\sin 3\theta}$$

$$= \frac{\sin 3\theta . \cos\theta}{\sin 3\theta} + \frac{\cos 3\theta . \sin\theta}{\sin 3\theta}$$

$$= \cos\theta + \sin\theta . \cot 3\theta$$

05. $\dfrac{\sin 3\theta + 2.\sin 5\theta + \sin 7\theta}{\sin\theta + 2.\sin 3\theta + \sin 5\theta} = \cos 2\theta + \sin 2\theta . \cot 3\theta$

LHS

$$= \frac{\sin 7\theta + \sin 3\theta + 2.\sin 5\theta}{\sin 5\theta + \sin\theta + 2.\sin 3\theta}$$

$$= \frac{2\sin\left(\frac{7\theta + 3\theta}{2}\right).\cos\left(\frac{7\theta - 3\theta}{2}\right) + 2.\sin 5\theta}{2\sin\left(\frac{5\theta + \theta}{2}\right).\cos\left(\frac{5\theta - \theta}{2}\right) + 2.\sin 3\theta}$$

$$= \frac{2\sin 5\theta \,.\, \cos 2\theta \;+\; 2.\sin 5\theta}{2\sin 3\theta \,.\, \cos 2\theta \;+\; 2.\sin 3\theta}$$

$$= \frac{2\sin 5\theta \,.\, (\cos 2\theta + 1)}{2\sin 3\theta \,.\, (\cos 2\theta + 1)}$$

$$= \frac{\sin 5\theta}{\sin 3\theta}$$

$$= \frac{\sin (3\theta + 2\theta)}{\sin 3\theta}$$

$$= \frac{\sin 3\theta \,.\, \cos 2\theta}{\sin 3\theta} + \frac{\cos 3\theta \,.\, \sin 2\theta}{\sin 3\theta}$$

$$= \cos 2\theta \;+\; \sin 2\theta \,.\, \cot 3\theta$$

06. $\dfrac{\cos 3A \;-\; 2.\cos 5A \;+\; \cos 7A}{\cos A \;-\; 2.\cos 3A + \cos 5A}$

$$= \cos 2A \;-\; \sin 2A \,.\, \tan 3A$$

LHS

$$= \frac{\cos 7A \;+\; \cos 3A \;-\; 2.\cos 5A}{\cos 5A + \cos A \;-\; 2.\cos 3A}$$

$$= \frac{2\cos\left(\frac{7A + 3A}{2}\right).\cos\left(\frac{7A - 3A}{2}\right) \;-\; 2.\cos 5A}{2\cos\left(\frac{5A + A}{2}\right).\cos\left(\frac{5A - A}{2}\right) - \;2.\cos 3A}$$

$$= \frac{2\cos 5A \,.\, \cos 2A \;-\; 2.\cos 5A}{2\cos 3A \,.\, \cos 2A \;-\; 2.\cos 3A}$$

$$= \frac{2\cos 5A \,.\, (\cos 2A - 1)}{2\cos 3A \,.\, (\cos 2A - 1)}$$

$$= \frac{\cos 5A}{\cos 3A}$$

$$= \frac{\cos (3A + 2A)}{\cos 3A}$$

$$= \frac{\cos 3A \,.\, \cos 2A}{\cos 3A} \;-\; \frac{\sin 3A \,.\, \sin 2A}{\cos 3A}$$

$$= \cos 2A \;-\; \sin 2A \,.\, \tan 3A$$

07. $\dfrac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x \;-\; \cos x}$

$$= \operatorname{cosec} 2x - \cot 2x$$

LHS

$$= \frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x \;-\; \cos x}$$

$$= \frac{\sin 5x + \sin x - 2\sin 3x}{\cos 5x \;-\; \cos x}$$

$$= \frac{2\sin\left(\frac{5x + x}{2}\right)\cos\left(\frac{5x - x}{2}\right) \;-\; 2\sin 3x}{-\;2\sin\left(\frac{5x + x}{2}\right).\sin\left(\frac{5x - x}{2}\right)}$$

$$= \frac{2\sin 3x \,.\, \cos 2x - 2\sin 3x}{-\;2\sin 3x \,.\, \sin 2x}$$

$$= \frac{2\sin 3x\,(\cos 2x - 1)}{-\;2\sin 3x \,.\, \sin 2x}$$

$$= \frac{\cos 2x - 1}{-\sin 2x}$$

$$= \frac{1 - \cos 2x}{\sin 2x}$$

$$= \frac{1}{\sin 2x} \;-\; \frac{\cos 2x}{\sin 2x}$$

$$= \operatorname{cosec} 2x - \cot 2x$$

SOLUTION TO Q SET - 3

01. $$\frac{\sin A + \sin 2A + \sin 3A + \sin 4A}{\cos A + \cos 2A - \cos 3A - \cos 4A} = \cot A$$

$$\text{LHS} = \frac{\sin 3A + \sin A + \sin 4A + \sin 2A}{\text{Cos } A - \cos 3A + \cos 2A - \cos 4A}$$

$$= \frac{2\sin\left(\frac{3A+A}{2}\right).\cos\left(\frac{3A-A}{2}\right) + 2\sin\left(\frac{4A+2A}{2}\right)\cos\left(\frac{4A-2A}{2}\right)}{-2\sin\left(\frac{A+3A}{2}\right).\sin\left(\frac{A-3A}{2}\right) - 2\sin\left(\frac{2A+4A}{2}\right).\sin\left(\frac{2A-4A}{2}\right)}$$

$$= \frac{2.\sin 2A . \cos A + 2.\sin 3A . \cos A}{-2.\sin 2A . \sin(-A) - 2.\sin 3A . \sin(-A)}$$

$$= \frac{2.\sin 2A . \cos A + 2.\sin 3A . \cos A}{2.\sin 2A . \sin A + 2\sin 3A . \sin A}$$

$$= \frac{2.\cos A \cancel{(\sin 2A + \sin 3A)}}{2.\sin A \cancel{(\sin 2A + \sin 3A)}}$$

$$= \frac{\cos A}{\sin A} = \cot A = \text{RHS}$$

02. $$\frac{\sin x - \sin 3x + \sin 5x - \sin 7x}{\cos x - \cos 3x - \cos 5x + \cos 7x} = \cot 2x$$

$$\text{LHS} = \frac{\sin 5x + \sin x - \sin 7x + \sin 3x}{\cos x - \cos 5x + \cos 7x - \cos 3x}$$

$$= \frac{2\sin\left(\frac{5x+x}{2}\right).\cos\left(\frac{5x-x}{2}\right) - 2\sin\left(\frac{7x+3x}{2}\right)\cos\left(\frac{7x-3x}{2}\right)}{-2\sin\left(\frac{x+5x}{2}\right).\sin\left(\frac{x-5x}{2}\right) - 2\sin\left(\frac{7x+3x}{2}\right)\sin\left(\frac{7x-3x}{2}\right)}$$

$$= \frac{2.\sin 3x . \cos 2x - 2.\sin 5x . \cos 2x}{-2.\sin 3x . \sin(-2x) - 2.\sin 5x . \sin 2x}$$

$$= \frac{2.\sin 3x . \cos 2x - 2.\sin 5x . \cos 2x}{2.\sin 3x . \sin 2x - 2\sin 5x . \sin 2x}$$

$$= \frac{2.\cos 2x \cancel{(\sin 3x - \sin 5x)}}{2.\sin 2x \cancel{(\sin 3x - \sin 5x)}}$$

$$= \frac{\cos 2x}{\sin 2x} = \cot 2x = \text{RHS}$$

03. $$\frac{\sin x - \sin 5x + \sin 9x - \sin 13x}{\cos x - \cos 5x - \cos 9x + \cos 13x} = \cot 4x$$

$$\text{LHS} = \frac{\sin 9x + \sin x - (\sin 13x + \sin 5x)}{\cos x - \cos 9x + \cos 13x - \cos 5x}$$

$$= \frac{2\sin\left(\frac{9x + x}{2}\right).\cos\left(\frac{9x - x}{2}\right) - 2\sin\left(\frac{13x + 5x}{2}\right) . \cos\left(\frac{13x - 5x}{2}\right)}{-2\sin\left(\frac{x + 9x}{2}\right). \sin\left(\frac{x - 9x}{2}\right) - 2\sin\left(\frac{13x + 5x}{2}\right). \sin\left(\frac{13x - 5x}{2}\right)}$$

$$= \frac{2.\sin 5x . \cos 4x - 2.\sin 9x . \cos 4x}{-2.\sin 5x . \sin(-4x) - 2.\sin 9x . \sin 4x}$$

$$= \frac{2. \sin 5x . \cos 4x - 2.\sin 9x . \cos 4x}{2. \sin 5x . \sin 4x - 2\sin 9x . \sin 4x}$$

$$= \frac{2.\cos 4x\,\cancel{(\sin 5x - \sin 9x)}}{2.\sin 4x\,\cancel{(\sin 5x - \sin 9x)}}$$

$$= \frac{\cos 4x}{\sin 4x}$$

$$= \cot 4x = \text{RHS}$$

04. if $\sin 2x + \sin 6x = \cos 2x + \cos 6x$, Show : either $\tan 4x = 1$ OR $\cos 2x = 0$

$$\sin 6x + \sin 2x = \cos 6x + \cos 2x$$

$$2.\sin\left(\frac{6x + 2x}{2}\right). \cos\left(\frac{6x - 2x}{2}\right) = 2.\cos\left(\frac{6x + 2x}{2}\right). \cos\left(\frac{6x - 2x}{2}\right)$$

$$2.\sin 4x . \cos 2x = 2.\cos 4x . \cos 2x$$

$$\sin 4x . \cos 2x = \cos 4x . \cos 2x$$

$$\sin 4x . \cos 2x - \cos 4x . \cos 2x = 0$$

$$\cos 2x (\sin 4x - \cos 4x) = 0$$

$\cos 2x = 0$ OR $\sin 4x - \cos 4x = 0$

... PROVED

$$\sin 4x = \cos 4x$$

$$\frac{\sin 4x}{\cos 4x} = 1$$

$\tan 4x = 1$ ..... PROVED

05. if $\sin A + \sin 2A + \sin 3A = \cos A + \cos 2A + \cos 3A$,

Show : either $\tan 2A = 1$ OR $\cos A = -1/2$

$$\sin 3A + \sin A + \sin 2A = \cos 3A + \cos A + \cos 2A$$

$$2.\sin\left(\frac{3A + A}{2}\right).\cos\left(\frac{3A - A}{2}\right) + \sin 2A = 2.\cos\left(\frac{3A + A}{2}\right).\cos\left(\frac{3A - A}{2}\right) + \cos 2A$$

$$2.\sin 2A . \cos A + \sin 2A = 2.\cos 2A . \cos A + \cos 2A$$

$$\sin 2A. (2\cos A + 1) = \cos 2A . (2 \cos A + 1)$$

$$\sin 2A. (2\cos A + 1) - \cos 2A . (2 \cos A + 1) = 0$$

$$(2 \cos A + 1) (\sin 2A - \cos 2A) = 0$$

$2\cos A + 1 = 0$ OR $\sin 2A - \cos 2A = 0$

$2,\cos A = -1$ $\qquad$ $\sin 2A = \cos 2A$

$\cos A = -1/2$ $\qquad$ $\dfrac{\sin 2A}{\cos 4A} = 1$

$\tan 2A = 1$ ..... PROVED

06. $\sin 10^\circ + \sin 50^\circ - \sin 80^\circ + \sin 140^\circ = \sqrt{2} . \sin 25^\circ$

LHS =

$\sin 50^\circ + \sin 10^\circ + \sin 140^\circ - \sin 80^\circ$

$$= 2 \sin\left(\frac{50 + 10}{2}\right).\cos\left(\frac{50 - 10}{2}\right) + 2\cos\left(\frac{140 + 80}{2}\right).\sin\left(\frac{140 - 80}{2}\right)$$

$$= 2 \sin 30 . \cos 20 + 2\cos 110 . \sin 30$$

$$= 2 \sin 30\left(\cos 110 + \cos 20\right)$$

$$= 2 . \frac{1}{2} \ 2\cos\left(\frac{110 + 20}{2}\right).\cos\left(\frac{110 - 20}{2}\right)$$

$$= 2\cos 65 . \cos 45$$

$$= 2 \frac{1}{\sqrt{2}} \cos 65$$

$$= \sqrt{2} \sin 25 = \text{RHS}$$

07. $\cos 40^\circ + \cos 50^\circ + \cos 70^\circ + \cos 80^\circ = \cos 20^\circ + \cos 10^\circ$

LHS =

$\cos 40^\circ + \cos 50^\circ + \cos 70^\circ + \cos 80^\circ$

$= \cos 80^\circ + \cos 40^\circ + \cos 70^\circ + \cos 50^\circ$

$= 2\cos\left(\frac{80+40}{2}\right).\cos\left(\frac{80-40}{2}\right) + 2\cos\left(\frac{70+50}{2}\right).\cos\left(\frac{70-50}{2}\right)$

$= 2\cos 60 . \cos 20 + 2\cos 60 . \cos 10$

$= 2\cos 60 . \left(\cos 20 + \cos 10 \right)$

$= 2 \ \frac{1}{2}\left(\cos 20 + \cos 10 \right)$

$= \cos 20 + \cos 10$

SOLUTION TO Q SET - 4

01. $\dfrac{\sin 3A . \cos 4A - \sin A . \cos 2A}{\sin A . \sin 4A + \cos A . \cos 6A} = \tan 2A$

LHS $= \dfrac{2. \cos 4A . \sin 3A - 2. \cos 2A . \sin A}{2. \sin 4A . \sin A + 2. \cos 6A . \cos A}$

$= \dfrac{\sin(4A+3A) - \sin(4A-3A) - \left(\sin(2A+A) - \sin(2A-A)\right)}{\cos(4A-A) - \cos(4A+A) + \cos(6A+A) + \cos(6A-A)}$

$= \dfrac{\sin 7A - \sin A - \left(\sin 3A - \sin A\right)}{\cos 3A - \cos 5A + \cos 7A + \cos 5A}$

$= \dfrac{\sin 7A - \sin A - \sin 3A + \sin A}{\cos 3A - \cos 5A + \cos 7A + \cos 5A}$

$= \dfrac{\sin 7A - \sin 3A}{\cos 7A + \cos 3A}$

$= \dfrac{2\cos\left(\frac{7A+3A}{2}\right).\sin\left(\frac{7A-3A}{2}\right)}{2\cos\left(\frac{7A+3A}{2}\right).\cos\left(\frac{7A-3A}{2}\right)}$

$= \dfrac{2.\cos 5A . \sin 2A}{2.\cos 5A . \cos 2A} = \dfrac{\sin 2A}{\cos 2A} = \tan 2A$

02. $$\frac{\sin 8\theta.\cos\theta - \sin 6\theta.\cos 3\theta}{\cos 2\theta.\cos\theta - \cos 3\theta.\cos 4\theta} = \cot 5\theta$$

$$\text{LHS} = \frac{2.\sin 8\theta\ .\cos\theta - 2.\sin 6\theta.\cos 3\theta}{2.\cos 2\theta.\cos\theta - 2.\cos 4\theta.\cos 3\theta}$$

$$= \frac{\sin(8\theta+\theta) + \sin(8\theta-\theta) - \left(\sin(6\theta+3\theta) + \sin(6\theta-3\theta)\right)}{\cos(2\theta+\theta) + \cos(2\theta-\theta) - \left(\cos(4\theta+3\theta) + \cos(4\theta-3\theta)\right)}$$

$$= \frac{\sin 9\theta + \sin 7\theta - \left(\sin 9\theta + \sin 3\theta\right)}{\cos 3\theta + \cos\theta - \left(\cos 7\theta + \cos\theta\right)}$$

$$= \frac{\sin 9\theta + \sin 7\theta - \sin 9\theta - \sin 3\theta}{\cos 3\theta + \cos\theta - \cos 7\theta - \cos\theta}$$

$$= \frac{\sin 7\theta - \sin 3\theta}{\cos 3\theta - \cos 7\theta}$$

$$= \frac{2\cos\left(\frac{7\theta+3\theta}{2}\right).\sin\left(\frac{7\theta-3\theta}{2}\right)}{-\ 2\sin\left(\frac{3\theta+7\theta}{2}\right).\sin\left(\frac{3\theta-7\theta}{2}\right)}$$

$$= \frac{2.\cos 5\theta\ .\ \sin 2\theta}{-\ 2.\sin 5\theta\ .\ \sin(-2\theta)}$$

$$= \frac{2.\cos 5\theta\ .\ \sin 2\theta}{2.\ \sin 5\theta\ .\ \sin 2\theta} = \frac{\cos 5\theta}{\sin 5\theta} = \cot 5\theta$$

03. $$\frac{\sin 3\theta.\cos 5\theta - \sin\theta.\ \cos 7\theta}{\sin\theta.\ \sin 7\theta - \cos 3\theta.\cos 5\theta} = \tan 2\theta$$

$$\text{LHS} = \frac{2.\cos 5\theta.\sin 3\theta - 2.\cos 7\theta.\ \sin\theta}{2.\sin 7\theta\ .\ \sin\theta + 2.\cos 5\theta.\cos 3\theta}$$

$$= \frac{\sin(5\theta+3\theta) - \sin(5\theta-3\theta) - \left(\sin(7\theta+\theta) - \sin(7\theta-\theta)\right)}{\cos(7\theta-\theta) - \cos(7\theta+\theta) + \cos(5\theta+3\theta) + \cos(5\theta-3\theta)}$$

$$= \frac{\sin 8\theta - \sin 2\theta - \left(\sin 8\theta - \sin 6\theta\right)}{\cos 6\theta - \cos 8\theta + \cos 8\theta + \cos 2\theta}$$

$$= \frac{\sin 8\theta - \sin 2\theta - \sin 8\theta + \sin 6\theta}{\cos 6\theta - \cos 8\theta + \cos 8\theta + \cos 2\theta}$$

$$= \frac{\sin 6\theta - \sin 2\theta}{\cos 6\theta + \cos 2\theta}$$

$$= \frac{2\cos\left(\frac{6\theta + 2\theta}{2}\right) . \sin\left(\frac{6\theta - 2\theta}{2}\right)}{2\cos\left(\frac{6\theta + 2\theta}{2}\right) . \cos\left(\frac{6\theta - 2\theta}{2}\right)}$$

$$= \frac{2.\cos 4\theta . \sin 2\theta}{2.\cos 4\theta . \cos 2\theta}$$

$$= \frac{\sin 2\theta}{\cos 2\theta}$$

$$= \cot 2\theta$$

04. $$\frac{\cos 3A . \sin 9A - \sin A . \cos 5A}{\cos A . \cos 5A - \sin 3A . \sin 9A} = \tan 8A$$

LHS $$= \frac{2. \sin 9A . \cos 3A - 2. \cos 5A . \sin A}{2. \cos 5A . \cos A - 2. \sin 9A . \sin 3A}$$

$$= \frac{\sin(9A + 3A) + \sin(9A - 3A) - [\sin(5A + A) - \sin(5A - A)]}{\cos(5A + A) + \cos(5A - A) - [\cos(9A - 3A) - \cos(9A + 3A)]}$$

$$= \frac{\sin 12A + \sin 6A - [\sin 6A - \sin 4A]}{\cos 6A + \cos 4A - [\cos 6A - \cos 12A]}$$

$$= \frac{\sin 12A + \sin 6A - \sin 6A + \sin 4A}{\cos 6A + \cos 4A - \cos 6A + \cos 12A}$$

$$= \frac{\sin 12A + \sin 4A}{\cos 12A + \cos 4A}$$

$$= \frac{2\sin\left(\frac{12A + 4A}{2}\right) . \cos\left(\frac{12A - 4A}{2}\right)}{2\cos\left(\frac{12A + 4A}{2}\right) . \cos\left(\frac{12A - 4A}{2}\right)}$$

$$= \frac{2.\sin 8A . \cos 4A}{2.\cos 8A . \cos 4A}$$

$$= \frac{\sin 8A}{\cos 8A} = \tan 8A = \text{RHS}$$

SOLUTION TO Q SET - 5

01. $\cos^2 x + \cos^2(x + 120) + \cos^2(x - 120) = 3/2$

We Prove : $2\cos^2 x + 2\cos^2(x + 120) + 2\cos^2(x - 120) = 3$

$1 + \cos 2\theta = 2\cos^2\theta$

$$\begin{aligned}
\text{LHS} &= 2\cos^2 x + 2\cos^2(x + 120) + 2\cos^2(x - 120) \\
&= 1 + \cos 2x + 1 + \cos(2x + 240) + 1 + \cos(2x - 240) \\
&= 3 + \cos 2x + \cos(2x + 240) + \cos(2x - 240) \\
&= 3 + \cos 2x + 2 \cdot \cos\left(\frac{2x + 240 + 2x - 240}{2}\right) \cdot \cos\left(\frac{2x + 240 - 2x + 240}{2}\right) \\
&= 3 + \cos 2x + 2 \cdot \cos 2x \cdot \cos 240 \\
&= 3 + \cos 2x + 2 \cdot \cos 2x \cdot \frac{-1}{2} \\
&= 3 + \cos 2x - \cos 2x \\
&= 3
\end{aligned}$$

02. $\sin^2\theta + \sin^2(120 + \theta) + \sin^2(120 - \theta) = 3/2$

We Prove : $2 \cdot \sin^2\theta + 2 \cdot \sin^2(120 + \theta) + 2 \cdot \sin^2(120 - \theta) = 3$

$1 - \cos 2\theta = 2\sin^2\theta$

$$\begin{aligned}
\text{LHS} &= 2 \cdot \sin^2\theta + 2 \cdot \sin^2(120 + \theta) + 2 \cdot \sin^2(120 - \theta) \\
&= 1 - \cos 2\theta + 1 - \cos(240 + 2\theta) + 1 - \cos(240 - 2\theta) \\
&= 3 - \cos 2\theta - \cos(240 + 2\theta) - \cos(240 - 2\theta) \\
&= 3 - \cos 2\theta - \left[\cos(240 + 2\theta) + \cos(240 - 2\theta)\right] \\
&= 3 - \cos 2\theta - 2 \cdot \cos\left(\frac{240 + 2\theta + 240 - 2\theta}{3}\right) \cdot \cos\left(\frac{240 + 2\theta - 240 + 2\theta}{2}\right) \\
&= 3 - \cos 2\theta - 2 \cdot \cos 240 \cdot \cos 2\theta \\
&= 3 - \cos 2\theta - 2 \cdot \frac{-1}{2} \cdot \cos 2\theta \\
&= 3 - \cos 2\theta + \cos 2\theta \\
&= 3
\end{aligned}$$